A Structural Empirical Model of R&D, Firm Heterogeneity, and Industry Evolution*

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Abstract

This paper develops and estimates a dynamic industry equilibrium model of R&D, R&D spill-overs, and productivity evolution of manufacturing plants in the Korean electric motor industry from 1991 to 1996. Plant-level decisions for R&D, physical capital investment, entry, and exit are integrated in an equilibrium model with imperfectly competitive product market. We use a Simulated Method of Moments estimator to estimate the cost of R&D, the magnitude of the R&D spill-over, adjustment costs of physical investment, and the distribution of plant scrap values. The recent approximation method of Weintraub, Benkard and Van Roy (2007) is applied. Counterfactual experiments of two policies are implemented. Increasing the elasticity of substitution between products increases plant innovation incentives and the plant turnover. In contrast, a lower entry cost does not change industry productivity. Although the market selection effect is strengthened by higher firm turnover, the plant's incentives to invest in R&D are reduced.

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1 Introduction

A large empirical literature has documented substantial and persistent firm productivity heterogeneity even within narrowly-defined industries.[1] Theoretical models of industry dynamics by Jovanovic (1982), Hopenhayn (1992), and Ericson and Pakes (1995) have been developed to explain the patterns of individual firm size, success, and failure observed in longitudinal micro-level data. These existing theoretical models share a common feature: a stochastic process that changes a firm's productivity (or knowledge of its productivity) over time. This process of productivity evolution is a key component that drives the success and failure of individual firms and the overall evolution of industry structure.

In this paper, we study an important source of productivity evolution: the investment in R&D by individual firms. Specifically, using micro data for producers in the Korean electric motor industry for the period 1991 to 1996, we estimate how a firm's productivity is affected by its R&D investment and how the R&D investments are determined by industry competition. There exists strong empirical evidence that a firm's technological position does not just evolve exogenously. Griliches (1998) provides an extensive survey of the empirical literature linking own firm R&D spending, R&D spill-overs and productivity growth. We extend these previous studies by investigating the R&D decision of firms within a dynamic industry equilibrium model.

We contribute to the existing literature in several dimensions. First, it is widely observed that a large fraction of firms reporting no R&D activity in even high tech industries. We reconcile this observation with Gibrat's law by allowing firms to survive by imitation. Furthermore, solving the firm's dynamic optimization problem enables me to identify how firm investment, output, and exit decisions interact with its productivity change, which relates firm R&D directly to firm heterogeneity. Second, the equilibrium industry structure provides a natural link from individual firm performance to aggregate

[1]See Bartlesman and Doms (2000) for an excellent survey of the micro productivity literature.

industry productivity and output by two mechanisms: the "market selection" mechanism, which operates through resource re-allocation from low efficiency to high efficiency firms or through entry and exit, and the active "firm learning" mechanism, which operates through individual firm's productivity improvement over time. Finally, following estimation of the model parameters, we are able to evaluate how pro-competition policies affect firm R&D, physical investment, entry, exit, and industry aggregates quantitatively.

The idea of investigating firm R&D, inter-firm spill-overs, and the evolution of industry structure simultaneously dates back to Dasgupta and Stiglitz (1980). Under imperfect product market competition, firms maximize their value of continuation given expectations about the evolution of their own and competitors' states. This implies that a firm's learning effort and investment decision are endogenously shaped by the level of product market competition as well as by pressure from potential entrants.[2] Yet, few previous empirical studies have attempted to estimate such a dynamic equilibrium model.[3] My estimation builds on the industry dynamics model pioneered by Ericson and Pakes (1995). We adapt it to an environment with both physical-capital and knowledge-capital investments. The firms make entry, exit, and investment decisions each period and improve their productivity as a result of their own investments in knowledge capital. Furthermore, we allow for a technological spill-over from more productive to less productive firms. Finally, firms experience technological setbacks due to idiosyncratic exogenous shocks.

Ericson and Pakes (1995) propose a Markov Perfect Equilibrium concept to characterize the evolution of an industry. To circumvent the well-known heavy computational burden of the Markov Perfect Equilibrium, we use the "oblivious equilibrium" concept proposed by Weintraub, Benkard, and Van Roy (2007) to solve the industry equilibrium

[2] Spence (1984) shows that imperfect competition induces a free-rider problem in R&D effort given the existence of a spill-over. More recently, Bloom, Schankerman and Van Reenen (2004) use a panel of U.S. firms to empirically identify technological spill-over and product market rivalry.

[3] Some exceptions include Benkard (2004), Ryan (2005) and Collard-Wexler (2006), who use empirical dynamic oligopoly models to analyze industry pricing, industry performance, and optimal industry policy. Recently, Lentz and Mortensen (2005) estimate an equilibrium model of firm innovation developed by Klette and Kortum (2004) using a panel of Danish firms.

of the theoretical model. When there are large number of firms within the industry, the "oblivious equilibrium", which assumes that firms ignore current information about competitors states and condition their choices on the knowledge of the long run average industry state, closely approximates a Markov Perfect Equilibrium.

The model is used to study the process of R&D and productivity growth for producers in the Korean electric motor industry. In the first step, we utilize the model specification for static market competition to estimate the demand elasticity, returns to scale in production, and the process of plant level productivity. We also recover the plant's entry decision and new entrant's initial productivity distribution. In the second step, we use a Simulated Method of Moments estimator to estimate the dynamic investment model to recover the cost of R&D, the magnitude of the spill-over, adjustment costs of investment, and the distribution of plant scrap values. We apply a recent approach by Chernozhukov and Hong (2005), which is based on the Markov Chain Monte Carlo (MCMC) method, to obtain point estimates and confidence intervals. By explicitly controlling for imperfect competition, productivity heterogeneity and both physical-capital and knowledge-capital investment, the model is rich enough to reproduce the observed market structure and industry turnover patterns.

Furthermore, the estimation results show that each element of the model is critical in explaining the observed pattern in the data. The empirical results show that: first, a firm's own R&D effort improves its future productivity while this process is subject to substantial idiosyncratic uncertainty. The within-industry R&D spill-over is significant and helps to explain the observed producer R&D spending and productivity evolution patterns. On average, one dollar of competitor's R&D expenditure can substitute for 1.6 cents of own R&D input. Taking into account that the total R&D spill-over pool is much larger relative to any producer's own spending and the R&D spill-over is a public good, spill-overs are quite important to the less productive producers. Second, each producer also incurs substantial adjustment costs for physical capital investment. Third, the mean random scrap value and entry cost equals four years and six years of average firm profit, respectively. The relatively narrow hysteresis band, defined as the difference between the entry cost and the mean scrap value, explains the high turnover rate observed in the industry data. Finally, there is a complementarity between a firm's physical capital investment and its innovation incentives. Not only does a firm's own R&D investment, but also its physical capital investment, responds to the spill-over.[4]

Using the point estimates of the parameters, we implement counter-factual experiments to study the effects of two different pro-competitive policies. In the first experiment, the competitive pressure comes from the more elastic substitution between products within the industry. We investigate the case where there is a 5% reduction in the price-cost margin. In the second experiment, the entry cost is reduced to introduce 50% more entrants. As the simulations show, the two policies have very different implications for firm R&D effort, firm turnover, and industry productivity. Increasing the elasticity of substitution between products increases a firm's innovation incentive but slows firm turnover. In the long run, a 5% drop in price-cost margin improves industry productivity by 2.8%. On the other hand, lower entry cost doesn't change the industry productivity. Although the market selection is enforced by higher firm turnover, this is offset by a reduction in firm incentive to invest in R&D.

The paper is organized as follows. The following section highlights several interesting aspects of the data and motivates the modelling strategy. The related literature is also reviewed. The second section describes the economic environment and the industry equilibrium. The third section estimates and reports the model parameters. We implement counter-factual simulations of a set of policy changes and conclude in the final section.

[4]Bernstein and Nadiri (1989) report the same pattern using a dynamic duality approach.

1.1 Korean Electric Motor Industry

This paper will analyze a panel data set of Korean plants that manufacture electric motors (SIC31101 and SIC31102) from 1991 to 1996.[5] The data is from the Korean Annual Mining and Manufacturing Survey, which is collected by Korean Statistical Office for all the establishments with more than 5 workers on an annual basis. Environmental concerns have put energy efficiency on high priority for a lot of governments, including Korea. As an intermediate input sector, electric motor industry is important in this respect.

The majority of previous studies of R&D investment and knowledge spill-overs use data from Compustat or various kinds of R&D surveys, which usually include a limited number of firms competing in multiple industries.[6] Therefore, it is not a data set that meets the need of linking producer heterogeneity, industry structure and productivity dynamics. In our dataset, we observe a significant level of producer heterogeneity in size and capital intensity: the standard deviation of value-added shares in our sample is 4107.4, and the dispersion ratio of the 95 percentile over the 10 percentile is 39.88. Capital intensity (the ratio of capital over total value-added) has a standard deviation of 0.839, and the dispersion ratio of the 95 percentile over the 10 percentile is 36.78.

On the other hand, several features of the Korean electric motor industry data provide support for the estimation of our empirical model. First, each establishment reports detailed R&D expenditure on an annual basis, which provides us with a measure of its learning input. Second, electric motors is a manufacturing industry with a long history, mature technology and a large number of single-establishment firms.[7] Furthermore, the primary competition within this industry is based on producing motors with higher energy efficiency and lower cost. Firm R&D investment serves this purpose, and industry

[5] SIC31101 and SIC31102 is equivalent to NAICS 335312 (Motor and Generator Manufacturing) in U.S census. The establishments primarily engaged in manufacturing electric motors (except internal combustion engine starting motors), power generators (except battery charging alternators for internal combustion engines), motor generator sets (except turbine generator set units), and transformers.

[6] Griliches (1998) documents many studies using this data.

[7] On average 82% of the plants in the data are single-plant firms.

incumbents usually engage in in-lab process innovation. In our sample, 14.4% of firms have conducted active R&D investment, and among them the R&D intensity has a standard deviation of 0.387, and the dispersion ratio of 95 percentile over 10 percentile is 34.44.

Lastly, starting from 2000s, Korean Government is focusing on promoting knowledge-intensive industries such as Korean Electric Motor Industry. First, Korean Government is providing direct subsidy to R&D activities. For example, Korean Government is enacting tax deduction for facilities investments in research tests (Jung and Mah, 2013). Second, Korean Government is solidifying the foundation of R&D activities carried out by public research institutes, universities, and private companies, hence enhancing the communication between private and public institutes. Therefore, under such policy changes, it is important for us to learn how direct subsidy and R&D spillover will affect the aggregate R&D efforts and productivity in this industry.

The model we use in this paper is in part motivated by these observations. It intends to reconcile the observed producer heterogeneity in size, growth, decline, capital intensity and R&D intensity with the theory of optimizing agents. At the heart of the model is a stochastic process of individual producer's productivity, which is driven by past R&D investment and idiosyncratic shocks. Meanwhile each producer also invests in physical capital based on their expectations about the evolution of their own productivity and industry structure. Thus R&D expenditure, physical investment, exit and entry are the policy functions of each producer's dynamic optimization problem. Given some initial industry state, some producers grow stronger and gain market share, while others become weaker and finally exit the industry. An equilibrium is defined when the industry evolution is consistent with individual producer's perception.

1.2 Related Literature

There are three separate but relevant lines of literature that are related to this paper. The first builds the link between market structure and productivity. There have been two mechanisms explored by recent work. A series of papers by Aghion et al (2001, 2005 and 2006) emphasize this link by considering firm innovation effort as the primary source of productivity improvement.[8] Either market competition or entry pressure affects industry productivity through the change in each firm's R&D decisions. Using micro-level panel data from the UK, Aghion et al (2006) show that an entry threat spurs innovation incentives in sectors close to the technological frontier while in lagging sectors it discourages innovation. A second mechanism that has been emphasized in the literature is the impact of market competition on the process of resource reallocation among firms. Hopenhayn (1992) and Melitz (2003) are two examples. They abstract from the endogenous innovation considerations by taking firm productivity as exogenous. The primary mechanism they emphasize is the role of more intense market competition on resource allocation from less to more productive firms or the selection of more productive firms to stay in operation. On the empirical side, Foster, Haltiwanger, Krizan (2000) tests this mechanism using U.S. census data. Aw, Chung and Roberts (2004) use various two-digit industries from Korea and Taiwan and find that impediments to exit or entry can explain the difference in productivity and turnover patterns between the same industry in those two countries. Tybout (2000) reviews evidences from previous studies on developing countries. In summary, both mechanisms, "active learning" and "market selection" are confirmed to play a role. In our empirical model, both these mechanisms are present as a result of individual firm's optimization decisions.

The second line of literature explains and substantiates firm innovation efforts, especially the production experience and ideas the firms can obtain from each other freely *within* one specific industry. There have been quite a few game theoretical papers that

[8] Earlier works start with Dasgupta and Stiglitz (1980) and Aghion and Howitt (1992).

analyze firm technology innovation decisions when knowledge spill-overs are present. Specifically, Spence (1984) proposes a model of firms with Cournot competition in a homogeneous good. Each firm's cost depends on their accumulated industry knowledge. In a symmetric market equilibrium, he shows that with a positive spill-over, industry R&D is lower than the optimal level. However, as Jennifer Reinganum (1989) summarizes in her review article, "since it is largely restricted to special cases (e.g. deterministic innovations, drastic innovations, two firms, symmetric firms), this line of work has not yet had a significant impact on the applied literature in industrial organization; its usefulness for policy purposes should also be considered limited". In contrast, the empirical studies have found enormous heterogeneity across firms in their R&D decisions. Cohen and Klepper (1992), for instance, show that the distribution of firm R&D intensities within industries tends to be uni-modal, positively skewed, with a long tail to the right and to include a large number of non-performers. To match the empirical observations, it will be crucial to have a model which can accommodate firm-level heterogeneity in the investment decision. In terms of empirical studies, both Jaffe (1988) and Bloom, Schankerman and Van Reenen (2004) use Compustat and U.S. patent data to derive the significance of these effects from a set of reduced form regressions using patent output, R&D or Tobin'q as dependent variables.[9] They find a significant "strategic effect" from product market competition and a "spill-over effect" from firms in close technological space. However, these previous studies don't look at the firm's R&D decisions using a dynamic industry equilibrium framework. Thus they are not able to provide predictions on the effect of firm's innovation effort on industry productivity and evolution.

Finally, the empirical model of this paper draws heavily from the fully-dynamic workhorse industry evolution models by Hopenhayn (1992) and Ericson and Pakes (1995). In particular, I adopt a recent simplification of the computational algorithm by Weintraub, Benkard, and Van Roy (2007). Weintraub, Benkard and Van Roy (2007) develop an

[9] Jaffe (1988) used PICA database as additional source because product market information is provided by Compustat from 1993 onwards.

algorithm for computing an "oblivious equilibrium", in which each firm is assumed to make decisions based on its own state and knowledge of the long run average industry state. They further prove that, if the industry is not highly concentrated, as the market becomes large the oblivious equilibrium closely approximates the Markov Perfect Equilibrium defined in Ericson and Pakes (1995).

2 A Dynamic Model of R&D Investment

2.1 Sequence of Actions

In this section we extend the model of dynamic competition by Weintraub, Benkard and Van Roy (2007) to incorporate a knowledge spill-over and physical capital accumulation. Time is discrete and indexed by $t = 1, 2, 3, ..., \infty$. The firms within an industry are indexed by $i = 1, 2, 3, ...$ For each period t, each firm's state $\omega \in \Omega$ can be described by a pair of knowledge x and physical capital k, where x takes discrete values from the set $\mathbb{X} = \{x^1 < x^2 < x^3 <, ...\}$ and k takes discrete values from the set $\mathbb{K} = \{k^1 < k^2 < k^3 <, ...\}$. Accordingly $\Omega \in \mathbb{X} \times \mathbb{K}$ takes all the possible combinations of knowledge capital and physical capital, which is also a set of discrete values. The industry state at each period t is denoted s_t. Each of its element $s_t(\omega)$ is the total number of firms at state ω. The set of possible industry states is denoted by $\mathbb{S}$.

At the beginning of period t, all the incumbent firms engage in competition in the product market and simultaneously set their prices for each period t. A firm with state (x_t, k_t) earns profit $\pi(x_t, k_t; s_t)$. Then, incumbent firms and potential entrants make their exit and entry decisions simultaneously. Each incumbent firm observes an idiosyncratic scrap value ϕ_t, which is *i.i.d.* across different firms and time and has a well defined density function with support $\mathbb{R}_+$. Given the industry state s_t and its own state (x_t, k_t), the firm decides whether to exit. If it decides to exit, it can get the period profit plus the scrap value. The exit strategy $\chi_t = 1$ if a firm decides to exit and $\chi_t = 0$ otherwise. If it instead decides to remain in the industry, it can choose to invest in either knowledge capital or physical capital or both to improve its own state.

Meanwhile, there is also a pool of ex-ante identical potential entrants. To enter the industry, an entrant needs to pay a fixed amount of entry cost κ. Upon entry, the new entrants can draw their initial states from a distribution Φ^e. Each potential entrant compares its expected entry value with the entry cost to decide whether to become an incumbent next period. If it decides to enter, $\epsilon_t = 1$, otherwise $\epsilon_t = 0$. The number of firms entering at industry state s_t is a Poisson random variable, with mean $M(s_t)$.

In summary, the timing of events in each period is as follows:

1. Incumbent firms simultaneously set their prices to receive static period profit $\pi(x_t, k_t; s_t)$
2. Incumbent firms privately observe an idiosyncratic scrap value ϕ_t. If they exit, they get that value. If not, they make R&D and physical capital investment decisions.
3. Potential entrants decide whether to enter next period based on current industry state s_t. Each entrant pays a fixed entry cost κ.
4. Firms exit and receive their scrap values.
5. The investment outcome of each firm is realized, the new entrants enter and take their initial draws of x and k from a fixed distribution Φ_e.

2.2 Static Competition

We will first look at how firms interact with each other in the product market each period. We assume each firm within an industry has a standard Cobb-Douglas production function with returns to scale parameters γ.

$$q_t = exp(x_t)(l_t^{\alpha}(k_t)^{1-\alpha})^{\gamma}, \tag{1}$$

where q_t is the output of the individual firm. x_t captures how much a firm's knowledge lies behind the current frontier, so it has a maximum value of zero. k_t is physical capital input and l_t is labor input.

Each firm produces a differentiated product and each one of them faces a demand function such that

$$q_t = Q_t(p_t/P_t)^\eta = \frac{I_t}{P_t}(\frac{p_t}{P_t})^\eta \tag{2}$$

where p_t is the price set by the firm, while Q_t and P_t are the industry-level output and price index. Accordingly, I_t is defined as the industry market size at time t. This demand function is from the widely-used monopolistic competition model by Dixit and Stiglitz (1977). The parameter η captures the elasticity of substitution between different products. Notice that since there is a limited number of firms within one single industry, we follow Yang and Heijdra (1993) and assume that each firm's output decision influences the aggregate industry price index.[10]

Thus each period, a firm takes quasi-fixed factors (k_t, x_t), exogenous variable factor prices w_t, aggregate market price P_t as given and chooses variable inputs l_t to maximize its profit

$$\pi_t = p_t(I_t, P_t, q_t)q_t - w_t l_t. \tag{3}$$

We could rewrite this problem as

$$max_{l_t} P_t^{1+\frac{1}{\eta}} I^{-\frac{1}{\eta}} (exp(x_t)k_t^{(1-\alpha)\gamma})^{1+\frac{1}{\eta}} (l_t^{\alpha\gamma})^{1+\frac{1}{\eta}} - w_t l_t. \tag{4}$$

and the optimal labor decision is derived as

$$l_t^* = [\frac{w_t I^{\frac{1}{\eta}}}{(exp(x_t)k_t^{(1-\alpha)\gamma})^{1/\eta+1} P_t^{1+\frac{1}{\eta}} (1+1/\eta)\alpha\gamma}]^{\frac{1}{(1+1/\eta)\alpha\gamma-1}}. \tag{5}$$

In equilibrium, the normalized industry price index P_t is determined by the industry state s_t. Let $\varphi_t = exp(x_t)k_t^{(1-\alpha)\gamma}$ and $s_t(\varphi)$ be the number of firms whose $\varphi_t = \varphi$, then

$$P_t = I^{1-\alpha\gamma}(\frac{w}{(1+1/\eta)\alpha\gamma})^{\alpha\gamma}(\sum_{\varphi} s_t(\varphi)\varphi^\sigma)^{-\frac{1}{\sigma}}. \tag{6}$$

[10] In the empirical application, we assume $I_t \equiv I$ is taken as exogenous to the model and time invariant. There have been recent interest in the interaction between aggregate uncertainty and firm responses (i.e., Bloom (2006)), which is out of the scope the current model.

where $\sigma = \frac{1+\eta}{\eta-(1+\eta)\alpha\gamma}$. Finally, the equilibrium maximized profit for firm with individual state $\varphi_t = exp(x_t)k_t^{(1-\alpha)\gamma}$ is

$$\pi(\varphi_t, s_t) = I(1 - (1 + \frac{1}{\eta})\alpha\gamma)\frac{\varphi_t^\sigma}{\sum_\varphi s_t(\varphi)\varphi^\sigma}. \tag{7}$$

In summary, the profit of each firm only depends on their relative magnitude of knowledge capital x_t and physical capital k_t. Under competition the implied cost reductions show up in a decline of the aggregate price index.

2.3 Transition of States: Knowledge Production and Physical Investment

Each period, a firm can choose to invest in either knowledge capital or physical capital or both to improve its own state. There are major differences between the two types of investment. Consider a firm with state (x_t, k_t). The investment in physical capital has a deterministic outcome. It also involves an adjustment cost $c(k_t, k_{t+1})$. So a firm could directly choose the level of physical capital k_{t+1} for next period. In contrast, the improvement of knowledge capital involves uncertainty and depends on the firm's own R&D, competitor's R&D, and its current knowledge capital level.

For each period t, firms exert learning effort to keep up with or to close their gap with the current frontier technology. The input of knowledge production consists of two components. One part is the firm's own research and development $d_t = d(x_t, k_t; s_t)$, and the cost of knowledge production is $c_d \cdot d_t \cdot k_t^{d_k}$, where the per-unit cost is larger for larger firms, if d_k is larger than 0. The other part is the R&D spillover from other establishments competing in the same industry d_{-t}. So the transition of knowledge capital from x_t to x_{t+1} is determined jointly by d_t and d_{-t}, and also subject to uncertainty. We assume that within a narrowly defined industry, "technology space" and "product space" are reasonably well overlapping. Thus, the rival establishments' technological positions will affect a firm's knowledge production directly. In our model, the spillover effect for a firm with individual state (x_t, k_t) is defined as

$$\sum_{x>x_t}\sum_{k}\theta\frac{s_t(x,k)}{N_t}$$

Recall that s_t is the industry state at time t, and N_t is defined as the total number of incumbents at time t. This specification assumes that a firm gets the spillover only from the firms which have higher knowledge capital than it has. This brings a backward advantage for the incumbents who are far from the technological frontier.[11] The composite term entering the knowledge production takes the following form:

$$D_t = d(x_t, k_t; s_t) + \sum_{x>x_t}\sum_{k}\theta\frac{s_t(x,k)}{N_t}$$

This specification is related to Jovanovic and MacDonald (1994) in the sense that learning depends on the firm's state and actions, and on the state of the industry, including the distribution of know-how in use. [12] However, unlike Jovanovic and MacDonald (1994), in our model the decision to invest in R&D doesn't preclude the opportunity to get the knowledge externality. The effectiveness of R&D in improving the next period productivity also depends on how big the firm builds itself into. This is also consistent with the empirical observations that while total R&D investment increases with the size of the firm, the R&D intensity is independent of firm size. [13]

Furthermore, like Weintraub, Benkard and Van Roy (2007), there is an idiosyncratic exogenous depreciation shock each firm will suffer with probability δ each period. In reality, it could capture two risks faced by individual firms. The first one is the firm level "organizational forgetting" documented by Benkard (2004), which causes the production process less efficient. The second one is the possibility that an individual firm has difficulty keeping up with the improvement in the industry frontier such that its relative

[11] Using UK plant-level data, Grifflith, Redding, and Simpson (2005) shows that technology transfer plays an important role in productivity improvement of non-frontier establishment.

[12] Jovanovic and MacDonald (1994) show that in a competitive industry, imitation by the firms that lag behind the frontier force some convergence of technology among establishments as the industry matures.

[13] See Klette and Kortum (2004) for a nice summary of the patterns of R&D investment and their relationship with productivity.

position deteriorates. With all the pieces we described so far, we can now introduce the knowledge production function. Specifically, for $x_t = x^j \in \mathbb{X}$

$$x_{t+1} = \begin{cases} x^{j+1}, & \text{with probability } \frac{(1-\delta)D_t}{1+D_t}; \\ x^{j-1}, & \text{with probability } \frac{\delta}{1+D_t}; \\ x^j, & \text{with probability } \frac{1-\delta+\delta D_t}{1+D_t} \end{cases} \tag{8}$$

The firm needs to pay an extra investment cost $c(k_t, k_{t+1})$ for adjusting physical capital level from k_t to k_{t+1}. By normalizing the purchase price of capital $c_k = 1$, we specify the adjustment cost of each establishment as:

$$c(k_t, k_{t+1}) = c_a (i_t/k_t)^2 k_t, \tag{9}$$

where $i_t = k_{t+1} - (1-\delta_c)k_t$ is the investment (divestment), c_a is the parameter for the component of convex cost of adjustment.[14]

2.4 Incumbent's Maximization Problem

Given the knowledge production technology described in the last section, for an establishment with (x_t, k_t) the value of continuation $V_c(x_t, k_t; s_t)$ is given by

$$\begin{aligned} V_c(x_t, k_t; s_t, \phi_t) = max_{d_t, k_{t+1}} (1-\xi) \cdot \{-c_d d_t k_t^{d_k} - c_k(1 - \mathbb{1}_{\{i_t<0\}} \cdot \wp) \cdot i_t \\ -c(k_t, k_{t+1}) + \beta E_{s_{t+1}}[V(x_{t+1}, k_{t+1}; s_{t+1}) | x_t, d_t, d_{-t}, s_t]\} + \xi \cdot \phi_t, \end{aligned} \tag{10}$$

where $d_t(x_t, k_t; s_t)$ and $k_t(x_t, k_t; s_t)$ are associated policy functions. c_d is the cost of per unit of R&D input, and d_k is the parameter of r&d cost w.r.t. capital size. One caveat is that there is an asymmetry of divestment (by the degree of $\wp$).

Let $V(x_t, k_t; s_t)$ be the establishment's value at the beginning of the current period. Each period, each incumbent establishment decides whether to stay or exit, so

$$V(x_t, k_t; s_t) = \pi^*(x_t, k_t; s_t) + E_{\phi_t}[max\{V_c(x_t, k_t; s_t, \phi_t), \phi_t\}], \tag{11}$$

[14] The presence of irreversibility is emphasized by Abel and Eberly (1996). We also estimated the model using a slightly different version that incorporated a parameter for resale cost. All our results are robust with this change.

where ϕ_t is the scrap value, which is assumed to be a random variable with distribution $U[0, u_b]$, and ξ is an exogenous exit rate. The incumbent's decision rule $\chi(x_t, k_t; \phi_t, s_t) = 1$ if it decides to exit, $\chi(x_t, k_t; \phi_t, s_t) = 0$ otherwise.

2.5 Entrant's Problem

The potential entrants are ex-ante identical. Upon entry, they draw their initial endowment of x and k. It is assumed that each period's exogenous technological progress is embodied in the new generations of potential entrants to the industry. So their *relative* technological positions are drawn from a time invariant distribution Φ^e. Each potential entrant incurs entry cost κ. Potential entrants' decision $\epsilon = 1$ if

$$V_e(s_t) \equiv \beta E_{s_{t+1}}[\int V(x_e, k_e, s_{t+1}|a, \epsilon) d\Phi^e | s_t] \geq \kappa. \tag{12}$$

The mass of entrants for time t is a poisson random variable with mean $M(s_t)$.[15] The new entrants give additional competitive pressure to the incumbents' improvement. Any incumbent who can not keep up with the technological frontier's movement is going to be finally driven out of the business by superior "new generations".

2.6 Equilibrium

Following Ericson and Pakes (1995) and Weintraub, Benkard and Van Roy (2007), we define symmetric markov perfect strategies to be an action denoted by $a \in \mathbb{A}$ and entry decision $\epsilon \in \Lambda$. Specifically in our case, $a = \{d, k, \chi\}$, where $d : \Omega \times \mathbb{S} \times \mathbb{R}_+ \to R_+$ is each firm's R&D investment strategy, $k : \Omega \times \mathbb{S} \times \mathbb{R}_+ \to \mathbb{K}$ is its physical investment strategy and $\chi : \Omega \times \mathbb{S} \times \mathbb{R}_+ \to \mathbb{R}_+$ is its exit strategy. Similarly, define the entry strategy for potential entrants as: $\epsilon \in \Lambda : \mathbb{S} \to \mathbb{R}_+$. Define the value function $V(x, k, s|a', a, \epsilon)$ as

[15]The poisson random variable is justified by the following entry model: there are N potential entrants, $v_N(i)$ is the expected present value for each entering firm if i firms enter simultaneously. Each potential entrant employs the same strategy, the condition for a mixed strategy Nash Equilibrium is: $\sum_{i=0}^{N-1} C_{N-1}^i p_N^i (1-p_N)^{N-1-i} v_N(i+1) = \kappa$. The equation has a unique solution $p_N^* \in (0, 1)$, the number of firms entering is a binomial random variable Y_N with parameters (N, p_N^*). As $N \to \infty$, $Y_N \Rightarrow Z$, which is a poisson random variable with mean M.

the expected discounted payoffs for a firm at individual state (x, k) and industry state s playing strategy $a' \in \mathbb{A}$ while all rival firms follow strategy $a \in \mathbb{A}$ and potential entrants follow strategy ϵ. Then Markov Perfect Equilibrium strategies a and ϵ satisfy that:

1. for an incumbent $V(x, k, s|a, a, \epsilon) \geq V(x, k, s|a', a, \epsilon)$, $\forall a' \in \mathbb{A}$
2. entrants satisfy the zero profit condition such that $\beta E_{s'}[\int V(x, k, s'|a, \epsilon) d\Phi^e|s] \leq \kappa$, with equality if the mass of entrants $M(s) > 0$

2.7 Oblivious Equilibrium and Computation

In this section, we define an Oblivious Equilibrium. This equilibrium concept is based on Weintraub, Benkard and Van Roy (2007) who establish that when the number of establishments is large, oblivious strategies, which ignore current information about competitors states and are conditioned only on the knowledge of long run average industry state, can closely approximate a Markov Perfect Equilibrium. Let $\tilde{\mathbb{A}} \in \mathbb{A}$ and $\tilde{\Lambda} \in \Lambda$ be the set of oblivious strategies. Then for oblivious strategies $a = (d, k, \chi) \in \tilde{\mathbb{A}}$ and $\epsilon \in \tilde{\Lambda}$, the associated expected state of the industry in the long run is $S_{a,\epsilon}$. Define $\tilde{V}(x, k|a', a, \epsilon)$ as the expected payoff of an incumbent under the assumption that its competitors' state will be equal to $S_{a,\epsilon}$ in all future periods. Then oblivious equilibrium strategies a and ϵ satisfy, given self-generated $S_{a,\epsilon}$:

1. for an incumbent $\tilde{V}(x, k|a, a, \epsilon) \geq \tilde{V}(x, k|a', a, \epsilon)$, $\forall a' \in \mathbb{A}$
2. entrants satisfy the zero profit condition such that $\beta \int \tilde{V}(x, k|a, \epsilon) d\Phi^e \leq \kappa$, with equality if the mass of entrants $M > 0$

Weintraub, Benkard, and Van Roy (2007) proves that when incumbent's strategies and the entry rate function are oblivious, the industry state s_t is an irreducible, aperiodic and positive recurrent Markov Chain. Their key insight is that when there are a large number of firms and the market tends not to be concentrated, each individual firm can not benefit by unilaterally deviating to an optimal (non-oblivious) strategy by keeping track of the true industry state, *averaged over the invariant distribution of industry states*. In other words, in any industry state that has significant probability of occurrence, the oblivious strategy approximates the Markov Pefect Strategy.

The Oblivious Equilibrium provides an attractive alternative for our estimation and computation purpose. The Korean electric motor industry, which is already a detailed level 5-digit industry, still has on average 180 firms each year during our sample period. It's infeasible, and possibly unreasonable, to assume that each firm keeps track of industry state every single period and solves their optimization problem and computes the industry equilibrium. On the other hand, the model structure still allows for strategic interactions between heterogenous firms and can accommodate important industry aspects such as an imperfectly competitive product market and the R&D spill-overs across different producers.

Our procedure for calculating the equilibrium will follow the above definition closely. Given a set of parameters, the steps to compute the equilibrium are:

1. Initial guess of the mean number of entrants M
2. Initial guess of the average long run industry structure s_0
3. Solve the monopolistic competition equilibrium aggregate price P given s_0.
4. Solve the incumbent's maximization problem and recover their optimal investment policy and exit policy: $a = (d, k, \chi)$ given s_0.
5. Construct the transition matrix $T_{x,k,\chi}$ using the optimal policies. The long run average industry structure is calculated as $s_1 = M(I - T_{x,k,\chi})^{-1}\Phi^e$.
6. If $|s_0 - s_1|$ is not close enough, go back to step (2).
7. Check the free-entry condition of potential entrants. If it doesn't hold, go back to step (1).

3 Estimation of Model Preliminaries

3.1 Korean Electric Motor Industry Data

The Korean Annual Manufacturing Survey reports detailed information on each producer's physical investment, R&D investment, number of workers and value-added. In this section I will briefly review some key data patterns for the electric motor industry.

First, by tracking each firm over the years, we assign them into three groups: those who survive until the end of 1996 and increase their total value-added shares compared with the beginning of sample, those who survive until the end of 1996 but decrease their total value-added shares, and those who exit the sample before the end of 1996. Table 1 summarizes the productivity change and investment patterns for each of three groups. Plant productivity is systematically correlated with plant expansion, decline and exit. Notice that the change in plant productivity, measured by differences in log TFP from 1991 to 1996 for expanding plants, is 0.029. The change is significantly higher than that of the other two groups. The plants that expand also invest much more. After controlling for the scale of plants,the improving plants' investment to value-added ratio is on average 0.133, 50% more than those who exit and 300% more than those who deteriorate. We also report the average R&D investment and R&D intensity of each plant over the sample period.[16] There is a strong positive relationship between average R&D intensity and the change in productivity over the years. The R&D intensity of expanding plants is four times more than that of the other two groups.

Table 1: Success and Failure of 1991 Establishments*

(Standard Deviations in Parentheses)

[16] For plants that exited, until the year of their exit.

Different Life Path	Survive and Expand Output	Survive and Reduce Output	Exit by 96
LnTFP(end)-LnTFP91	0.029	0.021	-0.025
	(0.291)	(0.054)	(0.134)
Investment in Capital	668.790	48.954	80.459
	(1546.40)	(84.569)	(211.951)
Investment Intensity (I/Vadd)	0.133	0.031	0.085
	(0.152)	(0.027)	(0.171)
R&D	150.148	13.444	9.773
	(615.976)	(40.333)	(33.911)
R&D Intensity (R&D/Vadd)	0.027	0.006	0.006
	(0.093)	(0.017)	(0.02)

*R&D and investment in millions of won.

Second, a considerable fraction of producers report zero R&D expenditure.[17] The R&D performers account for only 15% of the total observations during the sample years. The average R&D expenditure is 39.27 million won per year.[18] The major components of it are the wages for R&D workers and materials for R&D. As in Figure 1 and consistent with many previous empirical findings, the distribution of R&D intensity is positively skewed.[19] Among R&D performers, R&D intensity is negatively correlated with size, measured using either labor or capital.

[insert Figure 1 here]

Table 2: Summary Statistics

[17]Doraszelski and Jaumandreu (2006) reports similar patterns of R&D expenditure using Spanish Manufacturing Survey data.

[18]The average exchange rate between won and U.S. dollar during the sample period is 786:1.

[19]For instance, Cohen and Klepper (1992) reports the R&D intensity in FTC Line of Business Data to be positively skewed, with a long tail extending to the right.

variable	mean	std.	1%	median	99%	N
R&D expenditure	24.02	203.15	0	0	598	2767
R&D expenditure/Value-added	0.01	0.13	0	0	0.25	2767
Physical capital	637.83	2871.55	0	78.96	15194.3	2767
Physical investment	113.49	636.86	−39	2.83	2209	2767
Value-added	1052.515	4107.37	29.19	247	19026	2767
Wage bills	429.37	1616.64	17.8	113.36	6503.98	2767

units of variables: millions of won

Third, we observe significant resource reallocation during the sample period. Physical investment at the establishment level, which includes net capital expenditures (purchase minus sales) on buildings, machinery/equipment, and transport vehicles, averages 173.45 million won per year. There is huge heterogeneity in plant investment decisions, ranging from -40 (1 percentile) to 6439 (99 percentile). The electric motor industry also features high turnover. The average annual entry rate of new establishments on average is .34 and the exit rate of incumbents is .28 over the sample years.

Finally, the data does not provide information on physical units of output. Establishments show large dispersion in their value-added and labor size. On the cost side, we also have the information on total material expenditure and total wage expenditure for each establishment per year. The ratio of total revenue and total cost is stable at approximately 1.22 over the years.

3.2 Estimating Production Function and Demand Elasticity

The first stage of estimation focuses on the static part of the model, the production function and the demand curve faced by the industry. The key parameters to be recovered from this stage are the demand elasticity η and the returns to scale γ. Furthermore, given consistent estimates of the production function, establishment level productivity for each time period can be constructed.

To be consistent with the theoretical model, we assume that labor is short-run variable input, while capital is quasi-fixed. The empirical production function for each establishment i at time t is assumed to take the following form

$$Q_{it} = L_{it}^{\alpha_{lit}} K_{it}^{\alpha_{kit}} exp(\alpha_0 + x_{it} + u_{it}) \tag{13}$$

where $\alpha_0 + x_{it}$ is the productivity and u_{it} is an i.i.d. idiosyncratic shock. Notice that α_{lt} and α_{kt} are not restricted to be constant over time. Equation (14) can be rewritten in logarithm terms as

$$q_{it} = \alpha_0 + \alpha_{lit} l_{it} + \alpha_{kit} k_{it} + x_{it} + u_{it} \tag{14}$$

Then we follow ACF(2015) in estimating the production function. Assume that a firm's material input demand at t is obtained from a non-parametric and invertible function $f_t(\cdot)$ where

$$m_{it} = f_t(x_{it}, k_{it}, l_{it})$$

Substitute this back into equation (15), we will get

$$q_{it} = \alpha_0 + \alpha_{lit} l_{it} + \alpha_{kit} k_{it} + f_t^{-1}(m_{it}, k_{it}, l_{it}) + u_{it}. \tag{15}$$

However, since real output is not observed, we will use the information from the demand structure to rewrite this equation in terms of deflated value-added.[20] The plant-level demand

$$Q_{it} = (Q_{It})(P_{it}/P_t)^{\eta}$$

can be written in logarithm terms as:

$$p_{it} - p_{It} = \frac{1}{\eta}(q_{it} - q_{It})$$

Combined with the fact that log total revenue is

$$r_{it} = p_{it} + q_{it}$$

[20] I will need a quite restrictive demand structure (Dixit-Stiglitz). Considering the large number of firms in the industry, it is a reasonable approximation. Katayama, Lu and Tybout (2005) provide an alternative based on a nested-logit demand model.

we have the p_{It} deflated revenue $\tilde{r}_{it}$ as:

$$\tilde{r}_{it} = (1 + \frac{1}{\eta})q_{it} - \frac{1}{\eta}q_{It} \tag{16}$$

Finally, substituting the production function for q_{it} we defined in equation (16), we get an equation linking deflated plant revenue and the production function, demand parameters and plant productivity:

$$\tilde{r}_{it} = \tilde{\alpha}_0 + \tilde{\alpha}_{lit}l_{it} + \tilde{\alpha}_{kit}k_{it} + \tilde{f}_t^{-1}(m_{it}, k_{it}, l_{it}) + \tilde{u}_{it}. \tag{17}$$

Then in the second stage, we assume the productivity evolves according to a Markov Process, where

$$\begin{aligned} x_{it} &= g(x_{it-1}, d_{it-1}) + \xi_{it} \\ &= m_0 + m_1 x_{it-1} + m_2 \log(\frac{D_{it-1}}{K_{it-1}}) + \xi_{it} \end{aligned} \tag{18}$$

Then we rely on following moment conditions to estimate the model:

$$E\left[\xi_{it}\begin{pmatrix} l_{it-1} \\ k_{it-1} \\ l_{it-1} \cdot k_{it-1} \end{pmatrix}\right] = 0 \tag{19}$$

The estimation results are reported in table 3. The estimated mark-up is approximately 1.0138.

Table 3: Production Function Parameters

(Standard Errors in Parentheses)

	ACF
$\tilde{\alpha}_l$	0.6403*
	(0.008)
$\tilde{\alpha}_k$	0.3735*
	(0.003)
m_0	0.4287*
	(0.002)
m_1	0.7947*
	(0.003)
m_2	0.0046*
	(0.000)

*significant at the 1% level

4 Estimation of Dynamic Parameters

Given the first stage estimates of production function coefficients $\tilde{\alpha}_0$, $\tilde{\alpha}_k$ and the demand elasticity η, we estimate the set of dynamic parameters $\Theta_0 = [d_k, c_d, c_a, u_b, \delta, \theta, \wp]$ in the second stage. The first parameter d_k is the cost parameter of r&d with respect to capital size, which captures the effect of capital size on r&d cost. c_d is the effectiveness of R&D inputs to improve plant knowledge capital. c_a is the physical capital adjustment cost. u_b is the parameter of plant's scrap value distribution. δ represents the idiosyncratic uncertainty of the change in plant-level knowledge capital. θ controls the size of R&D spillover effect, and $\wp$ captures the asymmetry of divestment of physical capital. Since the estimation involves solving a complicated dynamic industry equilibrium with no closed form solutions, we use the method of simulated moments approach, which minimizes a distance criterion between key moments from the actual data and the simulated data.

Recent empirical techniques have been proposed to estimate the dynamic industry equilibrium model without solving the equilibrium. Especially related to this study is the estimation procedure proposed by Bajari, Benkard and Levin (2006), which handles both continuous and discrete control variables. Their approach breaks the estimation into two stages. In the first stage, firm policy functions are recovered by regressing observed actions on the observed state variables. The probability distribution which defines the evolution of the state of the industry is also recovered at this stage. In the second stage, the structural parameters which make these observed policies optimal are estimated. The major breakthrough of their approach is to avoid the computational burden of Markov Perfect Equilibrium, with trade-off of the precise calculation of agent's value function and policy function.

We only have the plant-level data for one single industry over a six year period, while the possible state space is very large. It makes the sampling error of estimating the policy functions a major concern if we were to adopt the strategy of Bajari, Benkard and Levin (2006). On the other hand, the large number of firms and low industry concentration make the weaker notion of equilibrium-Oblivious Equilibrium especially attractive, since it is proved to be a good approximation of MPE in this case.

However, there is also a cost of using Simulated Method of Moments at this stage compared with Bejari, Benkard, Levin (2006). The industry evolution model we use is not proved to have a unique equilibrium in general. Although the profit function, production technology, and the entrant's initial distribution are estimated without computing the model, there is a risk that the computational algorithm might select a different equilibria than that observed in the data. On the other hand, since we are only focusing on one single industry, this problem is alleviated by matching a full set of moments on policies and transition of states from observed and simulated data, which allow the data to confirm the correct equilibrium. Nevertheless, the uniqueness of the computed equilibrium is a strong assumption.

4.1 Moments

In this section, we describe the set of data moments utilized and how they are relevant for the identification of key parameters. The sample we use to estimate the dynamic parameters is an unbalanced panel of plants in the electric motor industry at year 1991 and their subsequent annual observations through 1996. All the entrants in subsequent years are only used to construct a frequency estimate of the initial state distribution Ψ^e and are excluded from the construction of moments.

Table 5 report the moments we use in our estimation. The first set of moments captures the key features of optimal plant R&D investment behavior in equilibrium. The R&D investment cost c_d affects plant's R&D investment intensity. It is also the driving force behind the productivity improvement of R&D performers. On the other hand, the idiosyncratic shock δ is shaped by the proportion of positive r&d investors. Given its own technological position and expectation on industry productivity evolution, a plant makes an optimal decision on whether to stick with the "corner solution" of investing zero R&D. Thus, the fraction of R&D performers is affected by δ. Notice here that all moments of the change of firm productivity overtime are constructed conditional on survival. So they are also affected by the industry competition and turnovers.

The second set of moments described in Table 5 relate to the plant's physical investment behavior. Following Cooper and Haltiwanger (2005), the cost parameters c_a helps to capture the nonlinear relationship between plant-level investment and profitability. The level of the investment ratio and the fraction of plant's positve investment depend critically on its magnitude. Fraction of divestment identifies parameter $\wp$, which controls the asymmetry of divestment, and the covariance between adjusted investment and productivity level helps identify d_k, which determines the effect of capital on R&D expenditure. On the other hand, since we model firm's investment behavior within an industry equilibrium, the investment moments also help to pin down other key model parameters such as intensity of competition, technological spill-over and R&D costs. Recent empirical investment literature also emphasized the role of partial irreversibility in describing micro-level plant investment behavior, i.e. if the plant has to incur a significant loss by selling its existing asset, then it takes more caution to invest during periods of high productivity shocks. On the other hand, when there is a negative shock plants tend to hold on to capital. We estimated a slightly more complicated model of incorporating this feature and the estimation results on all other parameters turn out to be robust to this extension.

Thirdly, the long run exit pattern helps to identify the scrap value distribution parameter.[21]

Finally, we use the autocorrelation between productivity to identify parameter θ, which controls the R&D spillover effect: we simulate a sequence of $\{x_{it}\}$ from our structural model, and calculate the autocorrelation based on the simulated sequence of productivity. More specifically, we run the second-stage estimation of the production function

[21] Obviously, the other moments are also affected by the exit pattern of incumbent firms in an industry equilibrium setting.

on the simulated sequence of productivity, where

$$\begin{aligned}\hat{x}_{it} &= g(\hat{x}_{it-1}, \hat{d}_{it-1}) + \xi_{it} \\ &= \tilde{m}_0 + \tilde{m}_1 x_{it-1} + \tilde{m}_2 \log(\frac{\hat{D}_{it-1}}{\hat{K}_{it-1}}) + \xi_{it} \end{aligned} \tag{20}$$

Then we match the estimated coefficients $\tilde{m}$'s with the coefficients m's estimated from the data, by changing the spillover parameter θ.

Table 5: Key Data Moments

	Identification	Data
R&D Investment and Productivity Improvement		
fraction of R&D performers	pin down δ	11%
R&D intensity of performers (R&D/Value-added)	pin down c_d	0.06
std relative productivity level	pin down δ/c_d	0.30
Physical Capital Investmet		
mean investment ratio ($\frac{i}{k}_{it}$)	pin down c_a	.19
fraction of positive investment	pin down c_a	49.5%
fraction of divestment	pin down $\wp$	5%
Cov(i/K, X)	pin down d_k	0.08
Firm Turnover		
mean exit rate	pin down u_b	15%
Estimated Evolution Path of Productivity		
m_1 (coefficient of lagged productivity)	pin down θ	0.7879*
		(0.003)
m_2 (coefficient of $\frac{D_{it}}{K_{it}}$)	pin down d_k	0.0046*
		(0.000)

4.2 Empirical Implementation and Computation Details

The estimation of the dynamic parameters Θ_0 is implemented according to the following procedures. First, denote the set of data moments in table 5 as Γ^d, which is a 10 by 1 vector. Second, for a given set of parameters Θ, the industry equilibrium is solved and optimal policy functions for R&D expenditure, physical investment and survival (d^*, k^*, χ^*) are generated. Third, initialized by the observed data in 1991, we use the optimal policy functions to simulate the path for each plant. Notice that for a single simulation, each plant's behavior and subsequently their investment outcomes need to be updated simultaneously for each time period. We are able to calculate $\Gamma^s(\Theta)$ for simulation s. Finally, the simulated moments are defined as:

$$\Gamma^S(\Theta) = \frac{1}{S}\sum_{s=1}^{S}\Gamma^s(\Theta)$$

The MSM estimate $\hat{\Theta}$ minimizes the weighted distance between data and simulated moments:

$$L(\Theta) = min_\Theta[\Gamma^d - \Gamma^S(\Theta)]'W[\Gamma^d - \Gamma^S(\Theta)]$$

where W is a positive definite matrix.

In our numerical analysis, $\hat{W}$ is calculated from a bootstrap procedure: we randomly re-sample the data, and calculate the interested moments for each sample, then we obtain a variance-covariance matrix based on these bootstrap samples.

The discretized space of productivity $\mathbb{X} \equiv [x_{\min}, x_{\max}]$, and the range of $\mathbb{X}$ is determined by standard deviation of productivity in the data. The number of grid M_x in $\mathbb{X}$ affects R&D incentives, because R&D investment is instantaneous and non-accumulative in our model. For example, move from some value x_0 to $x_0 + \Delta x$ needs more steps hence more R&D investment, if M_x is larger. Therefore, R&D expenditure will be higher if M_x is larger, given $x_{\min}$ and $x_{\max}$ are fixed. In our numerical analysis, M_x is chosen to balance the R&D incentive and computation burden. The discretized space of capital $\mathbb{K} \equiv [k_{\min}, k_{\max}]$. $k_{\min}$ and $k_{\max}$ are chosen to match $x_{\min}$ and $x_{\max}$ based on firm's static profit maximization strategy, where $k_{\min} = \frac{\log(\frac{\sigma(1-\alpha)\gamma}{c_k+\delta_k})+\sigma x_{\min}}{1-\gamma\sigma(1-\alpha)}$ and $k_{\max} \approx \frac{\log(\frac{\sigma(1-\alpha)\gamma}{(1-\beta)(c_k+\delta_k)})+\sigma x_{\max}}{1-\gamma\sigma(1-\alpha)}$. The number of grid M_k in $\mathbb{K}$ is chosen in a way that $\forall k \in \mathbb{K}, \delta K \in \mathbb{K}$. Otherwise, the discretized $\mathbb{K}$ will generate "unwilling" investment or divestment.

The objective function $L(\Theta)$ is from a complicated dynamic problem, thus is non-smooth and with many local optima. To handle this, we apply a recent approach proposed by Chernozhukov and Hong (2005), which develops a class of Laplace Type Estimators (LTE) to circumvent this problem in the computation of the classical extremum estimators. Their estimation procedure focuses on LTE, which are functions of integral transformations of the criterion function $L(\Theta)$ and can be computed using Markov Chain Monte Carlo. In addition, we use their inference procedures to calculate confidence intervals from the quartiles of the Quasi-posterior distribution.

4.3 Estimation Results

Several of the model parameters are directly calculated from the data. We follow Cooper and Haltiwanger (2005) to set annual discounting rate β at 0.95 and annual rate of depreciation δ_c at 0.06. Labor's share parameter is calculated as the average of sample observation's labor expenditure share α_{lit}, which has a value of 0.65. Investment cost c_k is normalized to be 1. Depreciation rate of capital δ_c is set to be the average depreciation rate of capital 0.15 in the data. Discount rate β is 0.925, which is calculated from the interest rate of Korean in 1991 to 1996. η is -5. Finally, exogenous exit rate ξ is 0.02, which is calculated as the exit rate of firms with both productivity and capital above the 90 percentile of the industry in the data.

The following table reports the point estimates and their $5\%-95\%$ confidence interval. The point estimates are the mean of random draws from the posterior distribution using Markov Chain Monte Carlo. We use the M-H algorithm in Chernozhukov and Hong (2005) to derive the confidence interval for the point estimates.

Table 6: Dynamic Parameter Estimates

	Point Estimate	90% confidence interval
δ	0.72	$[0.43, 0.96]$
c_d	0.45	$[0.24, 0.67]$
θ	1.60	$[1.20, 1.97]$
d_k	0.30	$[0.07, 0.52]$
c_a	0.19	$[0.02, 0.34]$
$\wp$	0.47	$[0.06, 0.91]$
u_b	2.74	$[1.78, 3.79]$

By solving the industry equilibrium using the reported point estimates Θ, we can also infer the fixed entry cost κ using the model's free entry condition, equation (13). To further evaluate the overall fit of the estimation, we also report the simulated moments at the point estimates in Table 7.

Table 7: Model Fit

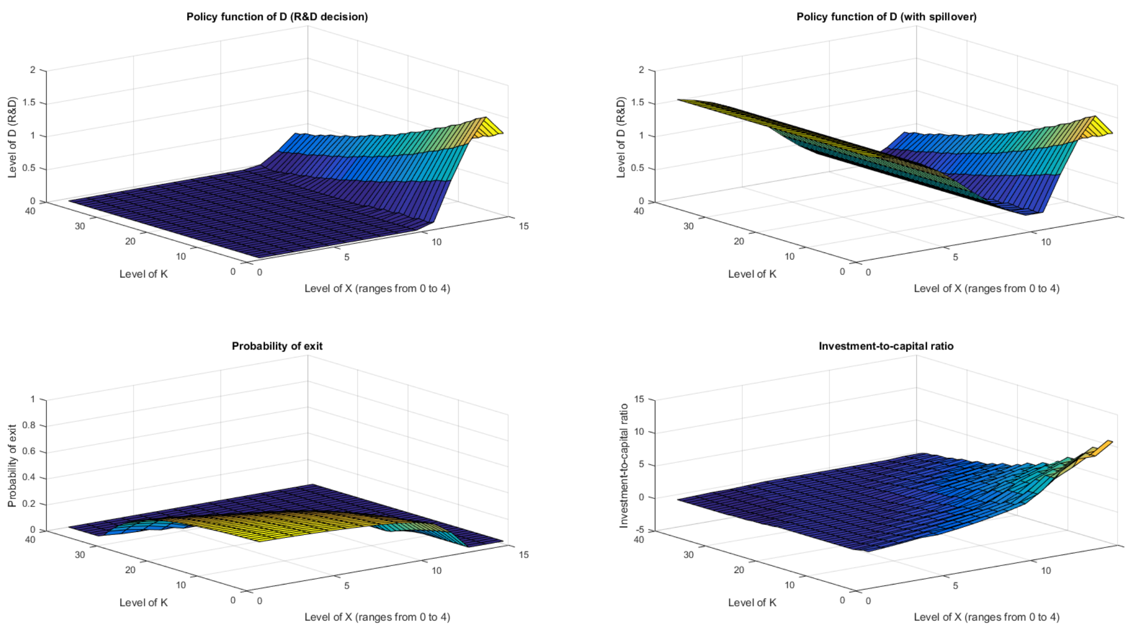
	Data	Simulation
R&D Investment and Productivity Improvement		
fraction of R&D performers	11%	10%
R&D intensity of performers (R&D/Value-added)	0.06	0.06
std relative productivity level	0.30	0.24
Physical Capital Investmet		
mean investment ratio ($\frac{i}{k}_{it}$)	.19	.19
fraction of positive investment	49.5%	54%
fraction of divestment	5%	5%
Cov(i/K, X)	0.08	0.07
Firm Turnover		
mean exit rate	15%	14%
Estimated Evolution Path of Productivity		
m_1 (coefficient of lagged productivity)	0.7879*	0.7975*
	(0.003)	(0.000)
	0.0046*	0.0054*
	(0.000)	(0.000)

Note: simulation is conducted $100,000$ rounds with burn-outs.

The simulated data does a good job of repeating the pattern of R&D investment and the productivity evolution, which is the core piece of this model.

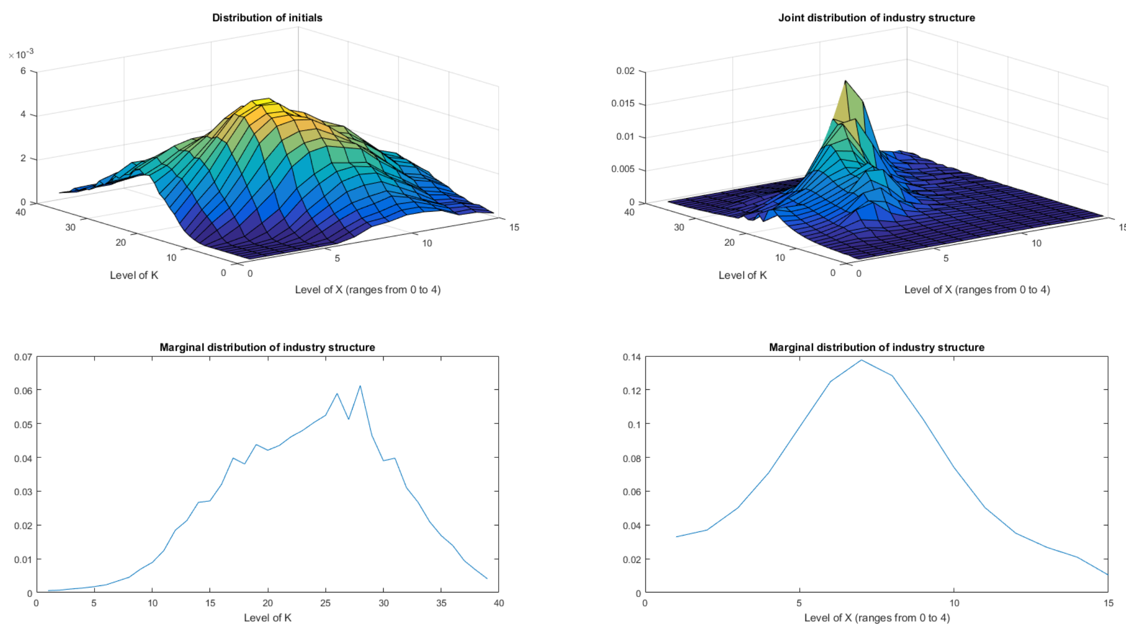
4.4 Evaluation of Results

The policy function derived from our simulation (with the estimated set of parameters) is shown as



The top-left panel shows that firms with higher productivity are more willing to make R&D investment, but higher capital level lowers R&D incentive because it brings higher R&D investment cost. The kink in the top-left panel is driven by the limit of maximum productivity a firm can achieve in the model. The top-right panel shows the summation of individual R&D activity and R&D spillover. From the top-right panel we can see that backward firms enjoy a significant amount of R&D spillover. which helps them move upward in the spectrum of productivity. The bottom-left panel shows the probability of exit of firms in the (productivity, capital) space: firms with both low productivity and low capital have the highest probability of exit, which is around 0.34. The probability of exit decreases when either productivity or capital increases. After reaching certain threshold in the (productivity, capital) space, firms will only suffer from possibility of exogenous exit, which is 0.02 in our model. The bottom-right panel shows investment-to-capital ratio. Firms with high productivity and low capital have the highest incentive to make investment, which have an investment-to-capital ratio near 8.9.

Figure below shows distribution of industry structure. The top-left panel shows the distribution of entrant in our model, which has the highest density in the middle, but smoothly distributed through the entire (productivity, capital) space. The top-right panel shows the distribution of industry structure in the equilibrium, from which we can observe that the industry is much more concentrated to states with middle-to-high level of capital, and middle level of productivity. The bottom-left and bottom-right panel shows the marginal distribution of capital and productivity in the equilibrium respectively.



How do the magnitude of the point estimates compare with previous studies? First, examine the R&D spill-over parameter θ, which has a value of 1.60. If we normalize the average R&D expenditure to 1 dollar, then a firm locating in the median of the spectrum of productivity will enjoy a 2.32 dollar R&D spillover from other firms. For a firm with lowest productivity in this industry, it will enjoy a 4.64 dollar R&D spillover from other firms. This may seem like a tiny amount at first glance, however, the total R&D spill-over pool is much larger relative to any producer's own spending. Taking into account that the R&D spill-over is a public good and affects the knowledge capital improvement of every producer, including those who do not perform R&D by themselves, spill-overs are important even though the magnitude of θ is small. Bloom, Schankerman and Van Reenen (2005) report that the private value of 1 dollar of spill-over is worth about 3 cents of own R&D in terms of the effect on market-value. Jaffe (1986) gets a similar number considering the effect on firm patents. Both of these studies use patent information to construct the "true" relevant R&D for each firm. Thus it is not surprising that their estimate of θ is about 2 times larger than what I find for the Korean electric motor industry. But without further information to link my data with Korean patent data, it's infeasible for me to narrow the range of firms in the pool like they do. Second, the idiosyncratic depreciation probability δ equals 0.72. It captures the two forces that erode the plant-specific knowledge, both because of the improvement of the industry frontier and because the loss of its own knowledge. The estimated R&D expenditure parameter of capital d_k is 0.3, which implies that higher capital results in higher unit-cost of R&D investment. Because d_k is less than one, it also indicates that incremental of R&D unit-cost is concave with respect to incremental of total capital.

In terms of quadratic adjustment cost, Cooper and Haltiwager (2005) reports a value of 0.225 while not controlling for fixed cost and 0.025 while controlling for fixed cost. Bloom (2006) reports a quadratic adjustment coefficient of 4.743 on a monthly basis, which implies a yearly value of 0.39. Our estimate of c_a, which equals 0.19, is slightly lower. It indicates that it is less costly to acquire or sell physical capital in the Korean electric motor industry. Our estimate of divestment parameter $\wp$ is 0.47, which implies that divestment will have a price discount of capital around 50%.

The estimated upper bound of scrap value distribution ub implies an unconditional mean of ? mil won, which is around four times of the industry average profit. On the other hand, the entry cost implied from free-entry condition is ? mil won. These values results in a quite narrow hysteresis band, which is driven by the high turnover rate observed in the data.

5 Policy Simulation: Competition, Innovation and Productivity

Koreas S&T policy is geared to acquiring core competences in strategic technology areas and developing an innovation system that will enable the nation to make a successful transition toward a knowledgebased economy. To achieve this policy goal, the Special Law for S&T Innovation was enacted in 1997. In accordance with the law, the Fiveyear Plan for S&T Innovation was launched in 1997. The plan contains specific plans for action to achieve the policy goal:

1. Corporate tax deduction of 50% of the increase in R&D and HRD investments over the annual average investments of the past four year or 5% if the current expenditures for the same purposes (15% for SMEs).

2. Corporate tax deduction of 5% of the total investment in equipment and facilities for R&D and/or HRD. Direct R&D subsidy for SMEs within KWR 100 million or 75% of the total investment.

In brief, the goal of this government policy is to increase R&D intensity in the Korean electric motor industry. However, because of the existence of R&D spillover, there is a trade-off between increasing aggregate R&D efforts and increasing aggregate productivity. More specifically, because of R&D spillover, firms with low productivity will have low incentive to invest in R&D activities because of their relative position in the spectrum of industry-wide productivity. Therefore, the impact of R&D subsidy will be different, depending on the different level of R&D spillover in this industry.

The goal of our policy simulation is to study how aggregate productivity and aggregate R&D efforts will respond to R&D subsidiary plans, given different level of R&D spillover effect. We will provide a menu of responses based on different policy simulations.

Using the structural estimates from previous section, we are able to simulate the industrial response from this policy change. Moreover, the oblivious equilibrium concept provides a nice way to summarize the average long run industry state, which is based on the equilibrium strategies of the plants. Statistics can be constructed from this long-run industry state to reflect the policy effect over a relatively long period. In addition, the industry evolution patterns before and after the policy change can also be simulated. Theses will highlight the short run transitory dynamic effects.

The first exercise is to examine by how much R&D spillover affects aggregate R&D efforts and hence dispersion of productivity among firms. With higher spillover effect θ, firms will have lower incentive to invest in R&D because now it is harder to differentiate from other firms by performing R&D. However, as a result dispersion of productivity will be smaller if the spillover effect θ is higher. To disentangle the effect of θ on dispersion of productivity and R&D incentives, we compare the benchmark case with another two cases: first, θ is increased by 50%, and R&D policy of firms is allowed to change endogenously. Second, θ is increased by 50%, but R&D policy is exogenously fixed to be the same as in the benchmark case. Table 10 summarizes the aggregate R&D efforts and dispersion of productivity.

Table 9: Effect of θ on R&D efforts and dispersion of productivity

	Aggregate productivity	Aggregate R&D efforts	Variance of productivity	productivity p90/p10 ratio
Benchmark	0.3492	24.8061	0.0725	5.4478
Case 1: High Theta	0.3519	21.4876	0.0636	5.246
Case 2: High Theta, exogenous R&D policy	0.3557	25.4621	0.0668	5.2502

From Case 1, we can see that a 50% increase of spillover effect θ result in a 0.8% increase of aggregate productivity, but also result in a 13.38% decrease of aggregate R&D efforts. By comparing Case 2 with Case 1, we can see that if the R&D policy is fixed unchanged when the spillover effect increases, the aggregate R&D efforts will actually increase, with an even further increase of aggregate productivity as in Case 1. However, the dispersion of productivity will also be larger in Case 2, as compared in Case 1.

Our counterfactuals target at reducing per-unit R&D expenditure c_d by 10%, 20%, 30%, 40%, and 50%, with government providing direct subsidy to R&D expenditure. Table 10 summarizes how aggregate R&D efforts and aggregate productivity will respond, under different scenarios when the R&D spillover in this market is either high or low.

TABLE 10.1: SOLUTION MENU OF POLICY SIMULATIONS, LOW θ

Counterfactual 1: low θ							
Cd		Aggregate productivity		Aggregate R&D efforts		Variance of productivity	productivity p90/p10 ratio
0.45	0	0.3492	0.00%	24.81	0%	0.073	5.40
0.405	10%	0.3531	1.12%	30.35	22%	0.077	5.43
0.324	20%	0.357	2.23%	38.31	54%	0.080	5.45
0.2268	30%	0.363	3.95%	47.20	90%	0.084	5.56
0.13608	40%	0.3699	5.94%	59.98	142%	0.090	5.79
0.06804	50%	0.3783	8.33%	79.35	220%	0.095	5.81

TABLE 10.2: SOLUTION MENU OF POLICY SIMULATIONS, HIGH θ

Counterfactual 2 high θ,(a 50% increase of θ as,compared in counterfactual 1)							
Cd		Aggregate productivity		Aggregate R&D efforts		Variance of productivity	productivity p90/p10 ratio
0.45	0	0.3519	0.00%	21.49	0%	0.064	5.20
0.405	10%	0.3558	1.10%	26.17	22%	0.066	5.24
0.324	20%	0.3597	2.23%	33.23	55%	0.070	5.41
0.2268	30%	0.3647	3.64%	42.56	98%	0.074	5.44
0.13608	40%	0.3721	5.73%	53.50	149%	0.081	5.44
0.06804	50%	0.3796	7.88%	72.12	236%	0.086	5.58

Comparing Table 10.1 with Table 10.2, we can see that with the same amount of per-unit R&D subsidiary, the level of aggregate productivity will be higher and dispersion of productivity will be lower if θ is higher. However, the level of aggregate R&D efforts will be lower if θ is higher. Therefore, depending on different objective of government R&D

policies, the role played by R&D spillover varies, and our counterfactual results provide a menu of how the market will react to the R&D subsidiary plan under different scenarios.

6 Conclusion

This paper develops and estimates a structural model of R&D investment and productivity evolution by manufacturing plants in the Korean electric motor industry from 1991 to 1996. Plant-level decisions on R&D investment, physical capital investment, entry, and exit are developed using an equilibrium industry evolution model. Plant productivity is affected by its own R&D and by spill-overs from the R&D of its competitors. The model provides a detailed set of pathways connecting R&D investment, plant productivity, plant physical investment and industry turnover patterns observed in the data.

The structural parameter estimates show that a plant's own R&D expenditure has a positive effect on its future productivity. There is also a small spill-over effect with one dollar of competitor's R&D expenditure substituting for 1.6 cents of own R&D input. The public externality of R&D is important given the large number of firms within the same industry. A narrow difference between the entry cost and the mean scrap value explains the high turnover rate in this industry. Finally, the industry equilibrium model provides a natural link from individual plant R&D decisions to aggregate industry productivity and output. This feature of the model provides us with a powerful tool to evaluate various industry or innovation policies. As our example experiments show: increasing the elasticity of substitution between products increases plant innovation incentives but slows plant turnover. In the long run, a 5% drop in price-cost margin improves industry productivity by 2.8%. On the other hand, a lower entry cost, which increases total entry by 50%, does not change industry productivity.

There are quite a few possible extensions to the current framework. An interesting one will be to look at the interaction of firm's decision to export, R&D, and the overall industry evolution. Given the fact that trade and innovation policy are considered to be among the most important institutional settings of emerging economies such as Korea, it will be important to provide a general framework to evalute how they interact and affect long run industry performance.

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7 Appendix

7.1 Profit from Static Competition

The static competition model of heterogenous firms is built on Melitz (2003). We assume that firm i within an industry has a standard Cobb-Douglas production function with returns to scale parameters γ. We will describe individual firm's problem by abstracting from the notation i for convenience.

$$q_t = exp(\bar{X}_t + x_t)(l_t^{\alpha}(k_t)^{1-\alpha})^{\gamma}, \tag{21}$$

where q_t is the output for firm i. Firm's efficiency is defined by its distance from the industry technological frontier $exp(\bar{X}_t)$. x_t captures how much a firm's knowledge lies behind current frontier, so it has a maximum value of zero. k_t is physical capital input and l_t is labor input. Furthermore, we assume $\bar{X}_t$ follows a deterministic exogenous process, which is determined by the world technological frontier.

Each firm produces a differentiated product and each one of them faces a demand function such that

$$q_t = Q_t(p_t/P_t)^{\eta} = \frac{I_t}{P_t}(\frac{p_t}{P_t})^{\eta} \tag{22}$$

where p_t is the price set by firm i, while Q_t and P_t are industry level output and price index. Accordingly, I_t is defined as the industry market size at time t. This demand function is from the widely-used monopolistic competition model by Dixit and Stiglitz (1977). The parameter η captures the elasticity of substitution between different products.

Thus each period, a firm takes quasi-fixed factors (k_t, x_t), exogenous variable factor prices w_t, aggregate market price P_t, and current frontier technology $exp(\bar{X}_t)$ as given and chooses variable inputs l_t to maximize its profit

$$\pi_t = p_t(I_t, P_t, q_t)q_t - w_t l_t \tag{23}$$

We could rewrite this problem as:

$$max_{l_t} P_t^{1+\frac{1}{\eta}} I^{-\frac{1}{\eta}} (exp(\bar{X}_t + x_t) k_t^{(1-\alpha)\gamma})^{1+\frac{1}{\eta}} (l_t^{\alpha\gamma})^{1+\frac{1}{\eta}} - w_t l_t. \tag{24}$$

Let's redefine industry price index as

$$\hat{P}_t = P_t exp(\bar{X}_t),$$

which allows us to write the optimal labor decision as

$$l_t^* = [\frac{w_t I^{\frac{1}{\eta}}}{(exp(x_t) k_t^{(1-\alpha)\gamma})^{1/\eta+1} \hat{P}_t^{1+\frac{1}{\eta}} (1+1/\eta)\alpha\gamma}]^{\frac{1}{(1+1/\eta)\alpha\gamma-1}}. \tag{25}$$

Let $\varphi_t = exp(x_t) k_t^{(1-\alpha)\gamma}$, $\sigma = \frac{1+\eta}{\eta-(1+\eta)\alpha\gamma}$, then $\frac{1}{(1+1/\eta)\alpha\gamma-1} = \frac{-\eta\sigma}{1+\eta}$. Substitute the optimal labor decision into the individual price equation $p(\hat{P}_t, q_t)$

$$\hat{p}(\hat{P}_t, \varphi_t) = p(\hat{P}_t, \varphi_t) exp(\bar{X}_t) = [\hat{P}_t^{(1+\frac{1}{\eta})(\alpha\gamma-1)} I^{\frac{1}{\eta}(1-\alpha\gamma)} (\frac{w}{(1+1/\eta)\alpha\gamma})^{\frac{1}{\eta}\alpha\gamma} \varphi_t^{-\frac{1}{\eta}}]^{\frac{-\eta\sigma}{1+\eta}}. \tag{26}$$

Furthermore $s_t(\varphi)$ is defined as the number of firms whose $\varphi_t = \varphi$. In equilibrium, normalized industry price index $\hat{P}_t$ is determined by the industry state s_t

$$\hat{P}_t = [\sum_{\varphi} s_t(\varphi) \hat{p}(\hat{P}_t, \varphi)^{1+\eta}]^{\frac{1}{1+\eta}}. \tag{27}$$

Substitute individual price equation $\hat{p}(\hat{P}_t, \varphi)$ into equilibrium industry price index

$$\hat{P}_t = I^{1-\alpha\gamma} (\frac{w}{(1+1/\eta)\alpha\gamma})^{\alpha\gamma} (\sum_{\varphi} s_t(\varphi) \varphi^{\sigma})^{-\frac{1}{\sigma}}. \tag{28}$$

Finally, we have the equilibrium maximized profit for firm with individual state $\varphi_t = exp(x_t) k_t^{(1-\alpha)\gamma}$ as

$$\pi(\varphi_t, s_t) = I(1 - (1 + \frac{1}{\eta})\alpha\gamma) \frac{\varphi_t^{\sigma}}{\sum_{\varphi} s_t(\varphi) \varphi^{\sigma}}. \tag{29}$$